# Robustness of End-to-End Detection in Coherent MAC-based Practicable Cooperative Networks

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***Abstract*—This paper considers a coherent multiple access channel (MAC)-based practicable cooperative wireless network consisting of non-ideal nodes that can be in error with a finite probability. Considering a practicable cooperative wireless network, this work obtains an optimal Neyman-Pearson (NP) criterion-based decoding statistic at the destination. The closed-form expressions of the probabilities of detection, false alarm, and symbol error are derived for the proposed detector, considering all possible network states to characterize the end-to-end detection performance of the practicable wireless network. The obtained probabilities are then averaged over the channel state information (CSI) to obtain the average probabilities. The practicable wireless networks with non-ideal nodes can significantly reduce the detection performance over the Genie/ error-free nodes. The work quantifies this performance difference via. the derived closed-form expressions and demonstrate a concurrence between the simulated and their derived analytical counterparts in the practicable cooperative networks.**

***Index Terms*—Cooperative wireless networks, hypothesis testing, coherent-multiple access channel (coherent-MAC).**

## I. Introduction

Cooperation in wireless networks like the Internet of Things (IoT) [1], [2], Body Area Networks (BANs), Cell-free Massive Multiple-input and Multiple-output (MIMO) [3], low-power wide area networks (LPWANs) and long-range WAN (LoRaWAN) [4], etc. that provide ubiquitous connectivity can utilize communication resources efficiently, combat multipath fading, achieving spatial diversity, resolving the difficulty to install multiple antennas, allow the cooperative nodes/ relays/ access points to collaborate for information transmission. Cooperative wireless networks [5], [6] enable nodes to assist the source when forwarding its information to the destination. Cell-free Massive-MIMO [3] and networks listed earlier, potential candidates for 6G, have similar network where cooperative nodes can be seen as access points. Among the several cooperation strategies in literature [6]–[10], this work focuses on two-hop cooperative wireless networks, where the cooperative nodes listen to the source signal and forward the decoded information to the destination.

The advantages of cooperation in wireless networks [5], [6] with ubiquitous connectivity depending mainly on the performance of cooperative nodes. Nearly all work in the literature considers an ideal/ error-free cooperating node or cooperative node knowing the decoded information is correct. Works such as [10]–[14] have discussed the effect of error in the cooperative node and demonstrated a degraded cooperative performance due to the error propagation at the destination. One prominent strategy to avoid performance degradation is node selection [10], [15], [16]. This work refrains from such strategies as they depend on the accuracy of the parameters, such as signal-to-noise ratio (SNR)/ channel state information (CSI) statistics between the source-to-nodes links and nodes-to-destination links, etc. Acquiring these parameters in ubiquitous connected wireless networks are challenging.

The CSI accuracy plays a vital role in node selection. Works in [9], [13], [15]–[17] assume to know the exact CSI between the source, nodes, and destination links. But works in [8], [18] consider knowing partial CSI statistics, and [10] knowing the first and second-order moments to obtain the symbol error rate (SER) performance. Cooperative detection [15], [19], symbol error rate (SER)/ outage probability analysis [9], [20], [21] for ideal cooperative nodes are mathematically tractable but do not reflect the true cooperation performance of the practicable network. Moreover, all works [6], [8]–[10], [13], [15]–[17], [19]–[22] discussed above consider an orthogonal wireless multiple access channels (MAC) between the cooperating nodes and the destination. The orthogonality is acquired using different time/ frequency/ code resources. Also, considering an orthogonal MAC eliminates multi-access interference (MAI) and simplify the performance analysis [13], [18], [23] but lacks efficiency in time/ frequency/ code resources, making the coordination between the cooperating nodes and destination are cumbersome.

Following the above conclusions, this work addresses the issue by accurately modelling the practicable wireless network, proposing a detection statistic and quantifying the detection performance trade-off. This work considers a coherent MAC-based two-hop practicable cooperative wireless network with $N$ nodes that decode and forward the source information to the destination. The system model considering non-ideal nodes that can be in error with a finite probability is described in section II. Section III presents an optimal Neyman Pearson (NP) criterion-based likelihood ratio test (LRT) statistic at the destination for the described scenario. For the proposed detection scheme, an end-to-end cooperation performance in terms of the probability of detection $\bar{P}_D$, probability of false alarm $\bar{P}_{FA}$, and symbol error probability $\bar{P}_e$ are derived considering the possible network states of the practicable cooperative network. Further in section III, the obtained probabilities are averaged over the CSI statistics between the cooperating nodes and destination to get the averaged probabilities of detection $P_D$, false alarm $P_{FA}$ and symbol error $P_e$. Section

IV presents simulation result comparisons of the performance trade-off between the scenarios with ideal (Genie) and non-ideal nodes to validate the concurrence of the simulation and the derived analytical results for the practicable cooperative communication networks. The following section presents the system description.

## II. System Model

Consider a source and a destination that communicate via $N$ cooperating nodes/ relays, as shown in Fig. 1. The direct link between the source and the destination is considered obstructed and remains unavailable, a well-known type-II relay model in 3GPP LTE-A. The source broadcasts the signal $x \in \mathbb{C}$ to the $N$ cooperative nodes. The nodes DF the received information to the destination over coherent MAC, i.e., nodes use same time-frequency resources. The cooperative node $i, 1 \leq i \leq N$, first listen to the source signal $x$ and upon precoding relays the source information to the destination, denoted as $u_i$. Realistically, the cooperative node can be in error and without knowing can relay the incorrect source signal. The error may be because of the adverse channel conditions between the source and the cooperating node, limitation of the decoding scheme at the node, noise, etc. Such errors at the cooperative node $i$ are characterised in terms of the probabilities of detection $(P_{D_i})$ and false alarm $(P_{F_i})$ [24] and are referred as a node in a practicable network. The error-free node $i$ with $P_{D_i} = 1, P_{F_i} = 0$ is called a Genie node or an error-free node. The decoded signal $\hat{x}_i$ at the cooperative node $i$ is corresponding to the transmission of the source signal $x$. The baseband received signal $y \in \mathbb{C}$ at the destination corresponding to the transmission of the $N$ nodes over coherent-MAC in the practicable cooperative wireless network is

$$y = g_1 u_1 + \cdots + g_N u_N + w, = \sum_{i=1}^{N} |g_i|^2 \hat{x}_i + w, \tag{1}$$

where $g_i$ is the flat faded channel coefficient between the cooperating node $i$ and the destination when $u_i = g_i^* \hat{x}_i$ in (1). The operation $f^*$ denotes complex conjugate of $f$. Noise $w \in \mathbb{C}$ is assumed to be circularly symmetric complex additive white Gaussian noise with zero mean and variance $\sigma^2$, i.e., $w \sim \mathcal{CN}(0, \sigma^2)$. Let there be $m$ out of $N$ nodes to be in error, i.e., $n = N - m$ nodes decode correctly. Equivalent system in (1) when $m$ out of $N$ nodes are in error is

$$y = \sum_{i=1}^{m} |g_i|^2 \hat{x}_i + \sum_{j=1}^{n} |g_j|^2 \hat{x}_j + w. \tag{2}$$

The first summation term in (2) corresponds to the set of nodes in error, i.e., $\hat{x}_i \neq x$. Similarly, the second summation term in (2) corresponds to the set of nodes decoding correctly, i.e., $\hat{x}_j = x$. The number of cooperative nodes $N$ are fixed and known. However, the number of nodes that decodes correctly $n$ and incorrectly $m$ may change. Note the set of nodes decoding correctly/ incorrectly may change even when the numbers $m$ and $n$ are fixed. Following section presents the

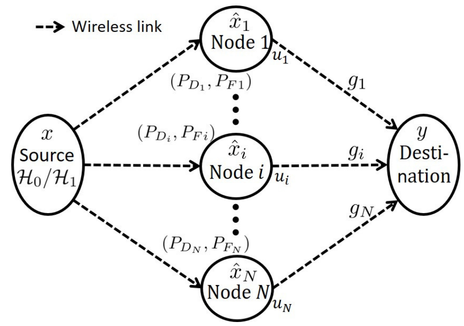


Fig. 1. Practicable Cooperative network with a source, a destination and $N$ non-ideal nodes.

cooperative method at the destination and characterizes the detection performance of the presented system.

## III. Detection Performance

Consider on-off signaling, i.e., $x \in \{0, 1\}$ with $x = 0$ and $x = 1$ denote the information source bits/ symbols corresponds to the null and alternative hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. For null hypothesis $\mathcal{H}_0$, $\hat{x}_i = 1$ and $\hat{x}_j = 0$ and for alternative hypothesis $\mathcal{H}_1$. The received signal $y$ at the destination (2) with on-off signaling is expressed as binary hypothesis testing

$$\mathcal{H}_0: \quad y = \sum_{i=1}^{m} |g_i|^2 + w \tag{3}$$

$$\mathcal{H}_1: \quad y = \sum_{j=1}^{n} |g_j|^2 + w. \tag{4}$$

For noise $w \sim \mathcal{CN}(0, \sigma^2)$ and known fading channel coefficients, the probability density function (PDF) of the received signal corresponding to the two hypotheses in (3) and (4) are $p(y; \mathcal{H}_0)$ and $p(y; \mathcal{H}_1)$, given as $p(y; \mathcal{H}_0) \sim \mathcal{CN}\left(b, \sigma^2\right)$ and $p(y; \mathcal{H}_1) \sim \mathcal{CN}\left(c, \sigma^2\right)$, with $b = \sum_{i=1}^{m} |g_i|^2$ and $c = \sum_{j=1}^{n} |g_j|^2$, respectively. The optimal NP criterion [24], to maximise the probability of detection for a stated probability of false alarm, the LRT-based decoding statistic at the destination is obtained by

$$\mathcal{L}(y) = \frac{p(y; \mathcal{H}_1)}{p(y; \mathcal{H}_0)} > \gamma', \tag{5}$$

where $\gamma'$ denotes the decision threshold. Solve the LRT in (5) using the PDFs obtained earlier to get the decoding statistic $T(y)$ at the destination in the coherent MAC-based practicable cooperative wireless networks with $N$ nodes for a given threshold $\gamma$ as

$$T(y) = y \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma. \tag{6}$$

The considered practicable cooperative network has nodes which can be in one of the two states, i.e., node may decode incorrectly $(l = 0)$ and correctly $(l = 1)$. Let $a_i^l$, for $1 \leq i \leq N$ and $l \in \{0, 1\}$ represent the state of node $i$. Thereby, $N$ nodes will have $2^N$ possible practicable network states. The network state vector **a** at a given time be denoted as

$$\mathbf{a} = [a_1^l, \cdots, a_i^l, \cdots, a_N^l]^T \in \mathcal{A}, \tag{7}$$

$$P_D = l\int_0^{\frac{\pi}{2}} \exp\left(\frac{-\gamma^2}{2\sigma^2\sin^2\theta}\right)\left(\frac{1}{\sigma^2\sin^2\theta}\right)^{-\frac{n}{2}}\left\{\frac{\sqrt{\pi}}{\Gamma(\frac{1+n}{2})}{}_1F_1\left(\frac{n}{2},\frac{1}{2};\frac{z^2}{2}\right)-\frac{\sqrt{2\pi}z}{\Gamma(\frac{n}{2})}{}_1F_1\left(\frac{1+n}{2},\frac{3}{2};\frac{z^2}{2}\right)\right\}d\theta \tag{23}$$

$$P_{FA} = k\int_0^{\pi/2} \exp\left(\frac{-\gamma^2}{2\sigma^2\sin^2\theta}\right)\left(\frac{1}{\sigma^2\sin^2\theta}\right)^{-\frac{m}{2}}\left\{\frac{\sqrt{\pi}}{\Gamma(\frac{1+m}{2})}{}_1F_1\left(\frac{m}{2},\frac{1}{2};\frac{z^2}{2}\right)-\frac{\sqrt{2\pi}z}{\Gamma(\frac{m}{2})}{}_1F_1\left(\frac{1+m}{2},\frac{3}{2};\frac{z^2}{2}\right)\right\}d\theta \tag{24}$$

$$\begin{aligned}P_e = 1-\int_0^{\pi/2} \exp\left(\frac{-\gamma^2}{2\sigma^2\sin^2\theta}\right)\Bigg\{k\left(\frac{1}{\sigma^2\sin^2\theta}\right)^{-\frac{m}{2}}\left\{\frac{\sqrt{\pi}}{\Gamma(\frac{1+m}{2})}{}_1F_1\left(\frac{m}{2},\frac{1}{2};\frac{z^2}{2}\right)-\frac{\sqrt{2\pi}z}{\Gamma(\frac{m}{2})}{}_1F_1\left(\frac{1+m}{2},\frac{3}{2};\frac{z^2}{2}\right)\right\}\\ +l\left(\frac{1}{\sigma^2\sin^2\theta}\right)^{-\frac{n}{2}}\left\{\frac{\sqrt{\pi}}{\Gamma(\frac{1+n}{2})}{}_1F_1\left(\frac{n}{2},\frac{1}{2};\frac{z^2}{2}\right)-\frac{\sqrt{2\pi}z}{\Gamma(\frac{n}{2})}{}_1F_1\left(\frac{1+n}{2},\frac{3}{2};\frac{z^2}{2}\right)\right\}\Bigg\}d\theta\end{aligned} \tag{25}$$

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where the set $\mathcal{A}$ contains all possible $2^N$ network states for $N$ nodes. The next result characterizes the performance of the decoding statistic obtained in (6) at the destination.

**Theorem 1.** *The instantaneous probabilities, i.e., the probability of detection ($\bar{P}_D$), probability of false alarm ($\bar{P}_{FA}$) and the probability of symbol error ($\bar{P}_e$), for the decoding statistic at the destination* (6) *when averaged over the network state* $\mathbf{a}$ *for the known fading coefficient in coherent-MAC-based practicable cooperative wireless network with $N$ nodes are*

$$\bar{P}_D = Q\left(\frac{\gamma-c}{\sigma}\right)\prod_{i=1}^{m}(1-P_{D_i})\prod_{j=1}^{n}P_{D_j} \tag{8}$$

$$\bar{P}_{FA} = Q\left(\frac{\gamma-b}{\sigma}\right)\prod_{i=1}^{m}P_{F_i}\prod_{j=1}^{n}(1-P_{F_j}) \tag{9}$$

$$\begin{aligned}\bar{P}_e = \frac{1}{2}\Bigg(1-Q\left(\frac{\gamma-c}{\sigma}\right)\prod_{i=1}^{m}(1-P_{D_i})\prod_{j=1}^{n}P_{D_j}\\ +Q\left(\frac{\gamma-b}{\sigma}\right)\prod_{i=1}^{m}P_{F_i}\prod_{j=1}^{n}(1-P_{F_j})\Bigg),\end{aligned} \tag{10}$$

*where the threshold $\gamma$, constants $b, c$, are given by* $\gamma = \sigma^2\frac{\ln\gamma'}{b-c}+\frac{b+c}{2}$, $b=\sum_{i=1}^{m}|g_i|^2$ *and* $c=\sum_{j=1}^{n}|g_j|^2$.

*Proof.* The instantaneous $\bar{P}_D$ defined in (11) is computed as

$$\begin{aligned}\bar{P}_D &\triangleq P(\mathcal{H}_1|\mathcal{H}_1) &(11)\\ &= P(T(y)>\gamma|\mathbf{a})P(\mathbf{a}|\mathcal{H}_1) &(12)\\ &= P(y>\gamma|\mathbf{a})\prod_{i=1}^{N}P(a_i^l|\mathcal{H}_1) &(13)\\ &= P(y>\gamma|\mathbf{a})\prod_{i=1}^{m}P(a_i^0|\mathcal{H}_1)\prod_{j=1}^{n}P(a_j^1|\mathcal{H}_1) &(14)\\ &= P(y>\gamma|\mathbf{a})\prod_{i=1}^{m}P(\hat{x}_i=0|\mathcal{H}_1)\prod_{j=1}^{n}P(\hat{x}_j=1|\mathcal{H}_1),\\ &= Q\left(\frac{\gamma-c}{\sigma}\right)\prod_{i=1}^{m}(1-P_{D_i})\prod_{j=1}^{n}P_{D_j}. &(15)\end{aligned}$$

where (12) follows from the cooperative network state defined in (7), (13) and (14) follows from the fact that the nodes are independent and can be in one of the two states $a_i^0$, $a_i^1$. The $Q$-function $Q(\cdot)$ in (15) denote the tail probability of the standard Gaussian PDF and as $p(y;\mathcal{H}_1)\sim\mathcal{CN}(c,\sigma^2)$. Similarly, the instantaneous false alarm, $\bar{P}_{FA}$, defined in (16) is acquired

$$\begin{aligned}\bar{P}_{FA} &\triangleq P(\mathcal{H}_1|\mathcal{H}_0) &(16)\\ &= P(T(y)>\gamma|\mathbf{a})P(\mathbf{a}|\mathcal{H}_0) &(17)\\ &= P(y>\gamma|\mathbf{a})\prod_{i=1}^{N}P(a_i^l|\mathcal{H}_0) &(18)\\ &= P(y>\gamma|\mathbf{a})\prod_{i=1}^{m}P(a_i^0|\mathcal{H}_0)\prod_{j=1}^{n}P(a_j^1|\mathcal{H}_0)\\ &= P(y>\gamma|\mathbf{a})\prod_{i=1}^{m}P(\hat{x}_i=0|\mathcal{H}_0)\prod_{j=1}^{n}P(\hat{x}_j=1|\mathcal{H}_0)\\ &= Q\left(\frac{\gamma-b}{\sigma}\right)\prod_{i=1}^{m}(1-P_{F_i})\prod_{j=1}^{n}P_{F_j}, &(19)\end{aligned}$$

where (17) and (18) follows from the network state vector defined in (7) and nodes states being independent. Considering the transmitted symbols $x\in\{0,1\}$ to be equally likely, the instantaneous probability of symbol error $\bar{P}_e$ for the practicable cooperative wireless network with $N$ nodes is

$$\begin{aligned}\bar{P}_e &\triangleq P(\mathcal{H}_0|\mathcal{H}_1)P(\mathcal{H}_1)+P(\mathcal{H}_1|\mathcal{H}_0)P(\mathcal{H}_0)\\ &= \frac{1}{2}(1-\bar{P}_D+\bar{P}_{FA}),\end{aligned} \tag{20}$$

where the probabilities hold their usual meanings, i.e., $P_D\triangleq P(\mathcal{H}_1|\mathcal{H}_1)$, $1-P_D\triangleq P(\mathcal{H}_0|\mathcal{H}_1)$, $P_{FA}\triangleq P(\mathcal{H}_1|\mathcal{H}_0)$, and $1-P_{FA}\triangleq P(\mathcal{H}_0|\mathcal{H}_0)$. The final expression of $\bar{P}_e$ in (10) is obtained using (15) and (19) in (20). □

The above probabilities in Theorem 1 consider the network states vector for the given wireless fading channel coefficients $g_i, 1\le i\le N$ between the cooperative node $i$ and the destination. The channel coefficient $g_i, 1\le i\le N$ follows a zero mean circularly symmetric complex Gaussian PDF with variance $\sigma_g^2$. Therefore, $b$ and $c$ are random variables defined by $b=\sum_{i=1}^{n}|g_i|^2$ and $c=\sum_{i=1}^{m}|g_i|^2$ to follow central

Chi-squared distributions with $2m$ and $2n$ degrees of freedom given as

$$f_B(b) \triangleq \frac{m^m b^{m-1}}{\sigma_g^m \Gamma(m)} \exp\left(\frac{-mb}{\sigma_g^2}\right), \quad b > 0 \tag{21}$$

$$f_C(c) \triangleq \frac{n^n c^{n-1}}{\sigma_g^n \Gamma(n)} \exp\left(\frac{-nc}{\sigma_g^2}\right), \quad c > 0, \tag{22}$$

where $\Gamma(\cdot)$ indicate Gamma function. Next, the probabilities obtained in Theorem 1 are averaged over the PDFs of the random variable in (21), (22) corresponding to the fading channel coefficients between the cooperating nodes and the destination.

**Theorem 2.** *The average probabilities of detection ($P_D$), false alarm ($P_{FA}$) and symbol error ($P_e$) for the detector* (6) *obtained at the destination when averaged over the network state* $\mathbf{a}$ *and the channel coefficients* $g_i, 1 \le i \le N$*, in coherent MAC-based practicable cooperative wireless network with* $N$ *nodes, given in* (23)*,* (24)*, and* (25)*, where* $k, l$ *are defined as*

$$k \;=\; \frac{m^m}{(2\sigma_g^2)^m \Gamma(m)} \prod_{i=1}^{m}(1 - P_{F_i}) \prod_{j=1}^{n} P_{F_j}, \tag{26}$$

$$l \;=\; \frac{n^n}{(2\sigma_g^2)^n \Gamma(n)} \prod_{i=1}^{m}(1 - P_{D_i}) \prod_{j=1}^{n} P_{D_j}. \tag{27}$$

*Proof.* The average probability of false alarm $P_{FA}$ defined in (28) is further solved as

$$P_{FA} \triangleq \int_{-\infty}^{\infty} \bar{P}_{FA} f_B(b) db \tag{28}$$

$$= \int_0^{\infty} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{(\gamma - b)^2}{2\sigma^2 \sin^2\theta}\right) d\theta \prod_{i=1}^{m} P_{FA_i} \prod_{j=1}^{n}(1 - P_{FA_j})$$

$$\frac{m^m b^{m-1}}{\sigma_b^m \Gamma(m)} \exp\left(-\frac{mb}{\sigma_b^2}\right) db \tag{29}$$

$$= k' \int_0^{\infty} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-(\gamma - b)^2}{2\sigma^2 \sin^2\theta}\right) b^{m-1} \exp\left(\frac{-mb}{\sigma_g^2}\right) d\theta db. \tag{30}$$

Use the Craig's result from (31) in (19) to obtain $\bar{P}_{FA}$. Use this $\bar{P}_{FA}$ and (21) in (28) to obtain (29).

$$Q\left(\frac{\gamma - b}{\sigma}\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{(\gamma - b)^2}{2\sigma^2 \sin^2\theta}\right) d\theta. \tag{31}$$

Rearrange (29) using $k' = \frac{m^m}{\pi \sigma_g^m \Gamma(m)} \prod_{i=1}^{m} P_{F_i} \prod_{j=1}^{n}(1 - P_{F_j})$ to get (30). Further solve (30) to separate the variables and interchange the integration order to get (32)

$$P_{FA} = k' \int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma^2}{2\sigma^2 \sin^2\theta}\right) \tag{32}$$

$$\underbrace{\int_0^{\infty} b^{m-1} \exp\left(\frac{-1}{2\sigma^2 \sin^2\theta} b^2 \cdot \frac{-(m \sin^2\theta - \gamma\sigma_g^2)}{\sigma^2 \sigma_g^2 \sin^2\theta} b\right) db\, d\theta}_{I_1},$$

$$= k \int_0^{\frac{\pi}{2}} \exp\left(\frac{-\gamma^2}{2\sigma^2 \sin^2\theta}\right) \beta^{\frac{-m}{2}} \exp\left(\frac{\delta^2}{8\beta}\right) D_{-m}\left(\frac{\delta}{\sqrt{2\beta}}\right) d\theta. \tag{33}$$

Use the standard integration result (34), from [25], over the integral $I_1$ in (32) to get (33), where $k$ was defined in (26).

$$\int_0^{\infty} x^{(m-1)} e^{(-\beta x^2 - \delta x)} dx$$

$$= (2\beta)^{\frac{-m}{2}} \Gamma(m) \exp\left(\frac{\delta^2}{8\beta}\right) D_{-m}\left(\frac{\delta}{\sqrt{2\beta}}\right), \tag{34}$$

for $\mathfrak{Re}(m) > 0$, $\mathfrak{Re}(\beta) > 0$. The symbol $\mathfrak{Re}(.)$ denotes the real part of the quantity. The parabolic cylinder function $D_{-m}\left(\frac{\delta}{\sqrt{2\beta}}\right)$ is defined as

$$D_{-m}\left(\frac{\delta}{\sqrt{2\beta}}\right) \triangleq 2^{-\frac{m}{2}} e^{-\frac{\delta^2}{8\beta}} \tag{35}$$

$$\left\{\frac{\sqrt{\pi}}{\Gamma\left(\frac{1+m}{2}\right)} {}_1F_1\left(\frac{m}{2}, \frac{1}{2}; \frac{\delta^2}{4\beta}\right) - \frac{\delta\sqrt{\pi}}{\sqrt{\beta}\Gamma\left(\frac{m}{2}\right)} {}_1F_1\left(\frac{1+m}{2}, \frac{3}{2}; \frac{\delta^2}{4\beta}\right)\right\},$$

where ${}_1F_1$ is hyper-geometric function [25, Section 9.14] given as

$${}_1F_1 = F(\alpha_1; \beta_1; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k}{(\beta_1)_k} \frac{z^k}{k!}.$$

Compare $I_1$ in (32) and (34) to get

$$\delta = \frac{m \sin^2\theta - \gamma\sigma_g^2}{\sigma^2 \sigma_g^2 \sin^2\theta}, \quad \text{and} \quad \beta = \frac{1}{2\sigma^2 \sin^2\theta}.$$

Let $k = k'\Gamma(m) 2^{-\frac{m}{2}}$ and use (35) in (33) to derive the expression of average probability of false alarm in (24). Using a similarly approach, the steps to obtain the average probability detection are

$$P_D = \int_{-\infty}^{\infty} \bar{P}_D f_C(c) dc, \tag{36}$$

$$= \int_0^{\infty} Q\left(\frac{\gamma - c}{\sigma}\right) \prod_{i=1}^{n} P_{D_i} \prod_{j=1}^{n}(1 - P_{D_j}) \frac{n^n c^{n-1}}{\sigma_g^n \Gamma(n)} \exp\left(\frac{-nc}{\sigma_g^2}\right) dc \tag{37}$$

$$= l' \int_0^{\infty} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{(\gamma - c)^2}{2\sigma^2 \sin^2\theta}\right) c^{n-1} \exp\left(\frac{-nc}{\sigma_g^2}\right) d\theta dc, \tag{38}$$

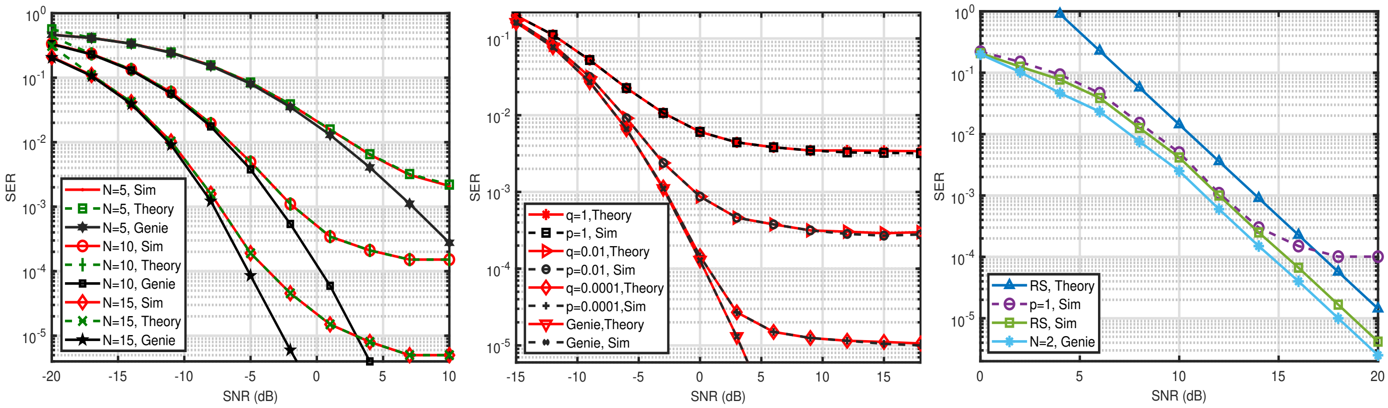
where (37) follows from substituting (31), (8) and (22) in (36). Use $l' = \frac{n^n}{\pi \sigma_g^n \Gamma(n)} \prod_{i=1}^{m}(1 - P_{D_i}) \prod_{j=1}^{n} P_{D_j}$ to simplify (37) to (38), and which is further rearranged to get (39)

$$P_D = l' \int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma^2}{2\sigma^2 \sin^2\theta}\right) \tag{39}$$

$$\underbrace{\int_0^{\infty} c^{n-1} \exp\left(\frac{-1}{2\sigma^2 \sin^2\theta} c^2 \cdot \frac{-(n \sin^2\theta - \gamma\sigma_g^2)}{\sigma^2 \sigma_g^2 \sin^2\theta} c\right) dc\, d\theta}_{I_2},$$

$$= l \int_0^{\frac{\pi}{2}} \exp\left(\frac{-\gamma^2}{2\sigma^2 \sin^2\theta}\right) \alpha^{-\frac{n}{2}} \exp\left(\frac{\xi^2}{8\alpha}\right) D_{-m}\left(\frac{\xi}{\sqrt{2\alpha}}\right) d\theta, \tag{40}$$

Substitute $l = l'\ \Gamma(n)\ 2^{-\frac{n}{2}}$, or use the $l$ as was defined in (26), and use the standard integration result in (34) over the




(a) (b) (c)

Fig. 2. SER vs. SNR plots with (a) varying number of cooperative nodes with a fixed probability of detection and false alarm, $\mathbf{P}_D = \mathfrak{L}(1 - p10^{-4}, 1 - p10^{-2}, N)$, $\mathbf{P}_F = q \times \mathfrak{L}(10^{-1}, 10^{-2}, N)$, $N \in \{5, 10, 15\}$ users, (b) varying probability of detection and false alarm with a fixed number of nodes/ users $N = 10$, where, $\mathbf{P}_D = \mathfrak{L}(1 - p10^{-4}, 1 - p10^{-2}, N), \mathbf{P}_F = q \times \mathfrak{L}(10^{-1}, 10^{-2}, N)$ with $p \in \{1, 10^{-2}, 10^{-4}\}$, $q \in \{1, 10^{-2}, 10^{-4}\}$, (c) $N = 2$ with a fixed value of $(\mathbf{P}_D = 1 - p10^{-5}, \mathbf{P}_F = p10^{-5})$ at each node.

integral $I_2$ in (39) to produce (40). Compare $I_2$ in (39) with the standard integration result (34) to obtain $\xi$ and $\alpha$, given as

$$\xi = \frac{n \sin^2\theta - \gamma\sigma_g^2}{\sigma^2\sigma_g^2 \sin^2\theta}, \quad \text{and} \quad \alpha = \frac{1}{2\sigma^2 \sin^2\theta}.$$

Expression for the average probability of detection (23) is obtained using the parabolic cylinder function definition from (35) in (40). Finally, use the averaged probabilities of detection and false alarm from (23) and (24) into the expression $P_e = \frac{1}{2}(1 - P_D + P_{FA})$ to get the average symbol error $P_e$ in (25) for the coherent MAC-based cooperative communication network with $N$ nodes. □

Next section presents important simulation observations.

## IV. Performance Analysis

This section illustrates the cooperative performance of the low to moderate number of nodes $N \in \{5, 10, 15\}$ nodes. Consider a scenario where the source, destination, and nodes have a single transmit and receive antenna. The nodes use coherent MAC channel to relay their observations to the destination where the coherent MAC-based fading channel coefficient follows a Rayleigh PDF and the phase follows a Uniform PDF. The noise is assumed to be additive white Gaussian with $w \sim \mathcal{CN}(0, \sigma^2)$. The fading channel coefficients of different users are assumed to be statistically independent. The nodes can be in error with a finite probability. An error-free node with zero probability of false alarm and unity probability of detection is called a *Genie node*. The cooperating node $i$ is characterized by the detection and false alarm probabilities pair, i.e., $(P_{D_i}, P_{F_i})$. The probabilities of detection for $N$ users evenly spaced between $(1-p10^{-4})$ and $(1-p10^{-2})$ with $0 \leq p \leq 1$ is denoted as $\mathbf{P}_D = \mathfrak{L}(1-p10^{-4}, 1-p10^{-2}, N)$, i.e., the vector $\mathbf{P}_D = [P_{D_1}, P_{D_2}, \cdots, P_{D_N}]$ has $N$ elements. The operation $\mathfrak{L}(f, g, N)$ produces a vector of size $N$ having values evenly spaced between $f$ and $g$. Similarly, probabilities of false alarm for $N$ users are taken to $\mathbf{P}_F = q \times \mathfrak{L}(10^{-1}, 10^{-2}, N)$ where the vector $\mathbf{P}_F = [P_{F_1}, P_{F_2}, \cdots, P_{F_N}]$ has $N$ elements. Fig. 2(a)-2(c) presents the probability of symbol error rate (SER) versus signal-to-noise ratio (SNR) comparisons.

Fig. 2(a) compares the SER performance on varying the number of nodes $N \in \{5, 10, 15\}$ for a fixed $(\mathbf{P}_D, \mathbf{P}_F)$ pair when $p = 1$ and $q = 1$. This figure compares the simulated (Sim) performance of the decoding statistic $T(y)$ in (6), the analytical (Theory) SER performance of the derived expression in (25), and the ideal nodes (Genie), i.e., when all the cooperating nodes have $(P_{D_i}, P_{F_i}) = (1, 0)$ for $1 \leq i \leq N$. It is evident from Fig. 2(a) that the SER simulation performance of the proposed non-ideal nodes based detection is in close agreement with its analytical derived expressions. Further, the simulation and theory plots at higher SER for the proposed scenario are in agreement with the Genie nodes scenario. At low SER, the practicable network scenario with non-ideal nodes deviates from the Genie nodes scenario. Moreover, the SER performance of the non-ideal nodes detector saturates. The relaying errors of the cooperative nodes account for the performance gap between the practicable and Genie nodes network scenarios. This is further reinforced by observing the effect of increasing the number of nodes. The SER saturation level improves by increasing the number of nodes.

Fig. 2(b) considers a fixed number of nodes $N = 10$ and different values of $(P_D, P_F)$ pair by selecting $p$ and $q$ values from the set $p \in \{1, 10^{-2}, 10^{-4}\}$, $q \in \{1, 10^{-2}, 10^{-4}\}$. The figure compares the detection performance of the simulation of the detection statistic in (6), analytical SER expressions in (25), and the scenario with Genie nodes. Fig. 2(b) follows the observations drawn from Fig. 2(a). Additionally, an improvement in the detection performance is observed by improving the probabilities of detection and false alarm values. Essentially, as the node behaviour closely resembles the Genie node, the SER performance of the nodes with practicable cooperative network tends to closely resemble the Genie nodes scenario. Hence, the proposed detector and the obtained analytical expressions for the practicable cooperative wireless network with $N$ nodes are comprehensive, and encompass the scenario with Genie nodes.

Fig. 2(c) considers $N = 2$ cooperative nodes with each having, $(P_D = 1 - p10^{-5}, P_F = p \times 10^{-5})$, same pair of probabilities. The figure compares the DF-based algorithm with node selection (RS, Simulation) and the upper bound on symbol error (RS, Theory), both from [15] along with the proposed non-ideal nodes based detection ($p = 1$, Simulation) in (6) using the above $(P_D, P_F)$ pair, and the scenario with Genie node (N=2, Genie). It can be observed that the node selection-based algorithm (RS, Simulation) [15] is in close agreement with the proposed practicable network detection at high SER. However, at lower SER the practicable network scenario deviates, as conferred from the previous figures. The performance improvement of the selection-based DF in [15] is due to the node's ability to participate only when they decode correctly, i.e., each node acting as a Genie node. The DF-based algorithm with node selection being ignorant of the practicable network, i.e., not accounting for the nodes, can be in error and have a constant inferior performance to the Genie node scenario.

## V. Conclusion

This paper considered a coherent MAC-based practicable cooperative wireless communication network that considered non-ideal nodes to send the source information to the destination. A decoding statistic was derived at the destination for $N$ nodes in a practicable cooperative network. The closed-form expressions for the end-to-end detection performance for the probabilities of detection, false alarm, and symbol error were derived considering the practicable wireless network states, which were further averaged based on the CSI statistics. The cooperative performance for practicable network obtained via simulation was in close agreement with their derived analytical counterparts. Further, the cooperation performance improves with the increase in the number of nodes or when the nodes act as Genie nodes. The proposed system framework, detection statistic, and the closed-form expressions for the practicable cooperative wireless networks is a generalization for the works with the cooperative wireless networks with ideal/ Genie nodes, i.e., later is a particular case of the presented work.

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